Linear Algebra & Dynamical Systems

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Course number: QTM2600

Meeting times: MW 1:45 -- 3:20PM

Meeting location: Gerber 102

Office hours: M 1:30 – 3:00pm, R 1:00pm – 3:00pm, or by appointment

Required Materials

Linear Algebra and Its Applications, fourth edition, Lay.

MATLAB

Other relevant references and assignments may be distributed through Blackboard.

Course Description

Linear algebra is perhaps the single most important collection of mathematical tools in common use today. It knits together topics that you may have already seen in some of your other courses, including solutions of systems of linear equations, linear programming and data fitting, to name a few. And for every topic you've seen, linear algebra generates many that you haven't. Over the course of the semester, we'll add several important tools to your analytic toolbox. For instance, we'll see that the line of best fit seen in QTM1300 and QTM2420 can easily be generalized to function of best fit, given some very basic assumptions. We'll also see how linear algebraic tools can be used in economic forecasting, discrete-time analysis, probabilistic modeling and many others.

General Course Policies

In general, you may use ANY outside aid on any assignment in the course. Allowed aids include but are not limited to another person, MATLAB, any calculator, etc. The student is responsible for his or her own knowledge; in particular, this means that it is completely reasonable to have a follow up oral component to any assessment in which I determine if you know what your submitted assignment seems to indicate you know.

Evaluation

The break down of the course grade is as follows:

Quizzes:	30%
Midterm:	30%
Final:	40%

Quizzes will be in-class, midterms will be take-home with an oral exam component. The final will likely be take-home+oral as well.

Honor Code

By handing in any assignment for evaluation of any kind in this course, you have certified the following statement:

I have neither given nor received unauthorized assistance on this assignment.

It is your responsibility as a member of the Babson community to understand the nature and implications of the Honor Code. Further information pertaining to Babson's Honor Code can be found in the undergraduate handbook.

Late and Makeup Policy

Makeups for quizzes and exams only will be granted in extreme circumstances (e.g., a death in your immediate family). Note that, for instance, booking a plane ticket to leave before the official start of vacation is not a valid reason to miss a quiz or exam.

Homework

As this is an upper-level elective course, homework will not be collected. That being said, it is absolutely vital that you complete the homework assignments. You will be responsible for all material presented in homework problems, even if this material was not mentioned specifically in class.

Accommodations and Disabilities

Any student who feels he or she may need an accommodation based on the impact of a disability should contact me privately and in a timely fashion to discuss his or her specific needs. Students must also contact the Manager of Disability Services at 781-239-4508 or in Hollister Hall to coordinate reasonable academic accommodations.

Religious Observances

Any student who faces a conflict between the requirements of this course and the observance of his or her religious faith should contact me as early in the semester as possible. In such an event, I will provide reasonable accommodations that do not unduly disadvantage the student.

Tentative Outline

The following is meant to be a general guide, not a definitive schedule. Topics may be added or omitted, and the time spent on each topic may be extended or contracted.

Date	Topic	Pre-class reading
23 Jan.	Introduction	
28 Jan.	Systems of Linear Equations	1.1, 1.2
30 Jan.	Solutions of Matrix Equations	1.4, 1.5
4 Feb.	Applications of Linear Equations	1.6
6 Feb.	Linear Independence, and More Applications	1.7, 1.10
11 Feb.	**Quiz 1**, Matrix Operations	2.1
13 Feb.	Invertible Matrices	2.2, 2.3
19 Feb.	Leontief Input-Output Model	2.6
20 Feb.	Leontief Price Equation	Handout
25 Feb.	**Quiz 2,** Markov Chains and Dynamical Systems	4.9, 5.6
27 Feb.	Eigenvectors and Eigenvalues	5.1
4 Mar.	PageRank	Handout
6 Mar.	Characteristic Equation	5.2
11 Mar.	Diagonalization	5.3
13 Mar.	**Midterm**	
18 Mar.	Spring Break	
20 Mar.	Spring Break	
25 Mar.	Vector Spaces	4.1
27 Mar.	Null spaces and column spaces	4.2
1 Apr.	Orthogonality	6.1, 6.2
3 Apr.	Orthogonal Projections and Least-Squares	6.3, 6.6
8 Apr.	Gram-Schmidt Process	6.5
10 Apr.	**Quiz 3**, Bases	4.3
15 Apr.	Patriot's Day	
17 Apr.	Coordinate Systems	4.4
22 Apr.	Change of Basis	4.7
24 Apr.	Singular Value Decomposition	7.4
29 Apr.	Principal Component Analysis	7.5
1 May	**Quiz 4,** Review	

Course Objectives

The course is broken up into 5 distinct but related modules. More than most subjects, linear algebra builds on itself, so it is important that the student masters the topics in one module in order to be well prepared to approach the next. All modules will involve the following general skills.

General skills

- Qualitatively reason and discuss linear algebraic concepts and applications
- Perform pen-and-paper algebraic computations using variables
- Perform computer-based algebraic computations using any tool of the student's choice
- Apply ideas developed in class to new situations and applications

Module 1: Vector algebra and linear transformations

- Understand the relationship between a system of linear equations and its matrix representation
- Compute the row-reduced echelon (RREF) form of a matrix and interpret the results
- Manipulate vector equations and understand the graphical interpretation of various vector operations
- Determine the number and nature of the solutions to a matrix equation
- Determine whether a collection of vectors is linear independent, and interpret the importance of linear independence in various contexts
- Determine whether a transformation is linear
- Write the matrix representation of a linear transformation with respect to a given basis

Module 2: Matrix algebra

Perform various matrix operations, including matrix-vector and matrix-matrix multiplication and transposition

- Understand the significance of matrix-matrix multiplication as linear functional composition
- Compute the inverse of an invertible matrix and perform matrix manipulations related to matrix inversion
- Understand the significance and applications of matrix inversion
- Understand the various interpretations of the determinant of a matrix and their connections
- Clearly identify the connection between the determinant of a matrix and the invertablility of a matrix

Module 3: Vector spaces, fundamental subspaces and bases

- Compute the null and column space of a linear transformation
- Understand the nature of dual spaces and how they relate to linear transformations
- Determine whether a collection of vectors forms a basis for a given vector space
- Compute the coordinates of a vector in any basis
- Write the change of basis matrix that converts the standard basis to a given basis and vice versa
- Understand the various connections between the dimensions of the null and column spaces of a matrix and the dimensions of various subspaces of the domain and range of the linear transformation

Module 4: Eigenvalues and eigenvectors

- Compute the eigenvalues and eigenvectors of a linear transformation
- Qualitatively reason about the meaning of eigenvalues and eigenvectors

- Construct and solve the characteristic equation of a matrix
- Diagonalize a matrix with real-valued eigenvalues

Module 5: Orthogonality, orthogonal projections and data fitting

- Understand inner products and orthogonality and their implications
- Construct an orthonormal basis from a given basis using the Gram-Schmidt process
- Perform orthogonal projections onto a given basis
- Use orthogonal projections for least-squares data fitting