Spontaneous oscillations in simple fluid networks

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krogh

> *In single capillaries the flow may become retarded or accelerated from no visible cause; in capillary anastomoses the direction of flow may change from time to time.*

(1922) Krogh

big questions

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- If so, what properties are necessary and/or sufficient to achieve oscillations?
- If not, are certain properties are inhibiting oscillations? Topology? Geometry? Viscosity?
- Or is biologic control simply essential for interesting behaviors to emerge?

related work

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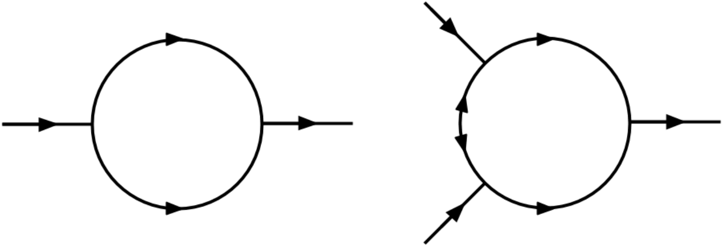
(2012) Forouzan *et al.* – *in vitro* experiments with real blood exhibit oscillations in good agreement with theoretical work

example networks and assumptions



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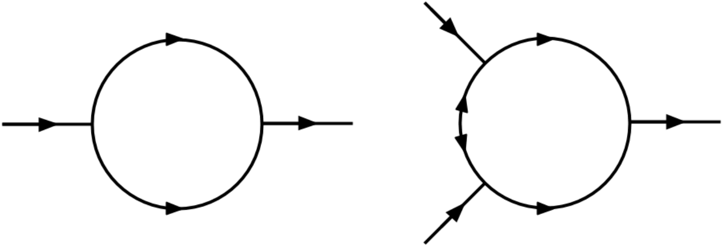

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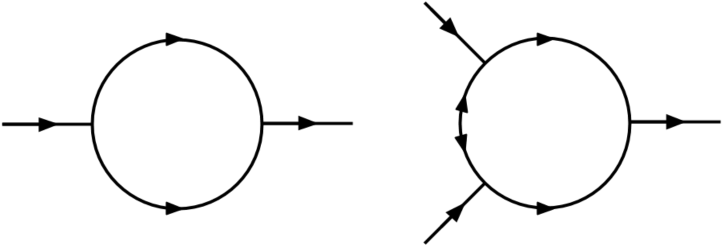

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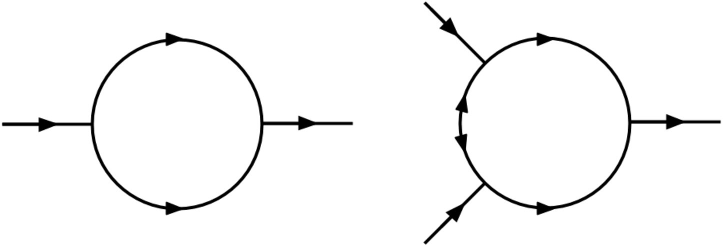

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- Inlet and outlet pressures and flows are constant
- Incompressible, laminar flow

viscosity

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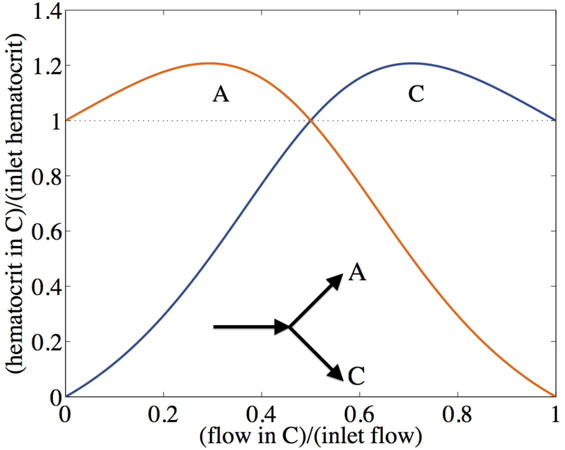
- $\phi \in [0, 1]$ is the hematocrit, *i.e.*, red blood cell concentration

plasma skimming

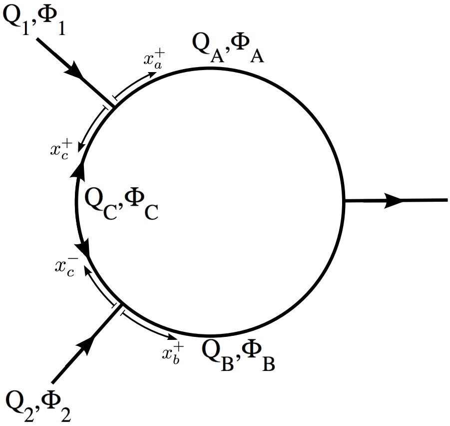


(1982) Klitzman and Johnson

video

H.N. Mayrovitz

current network



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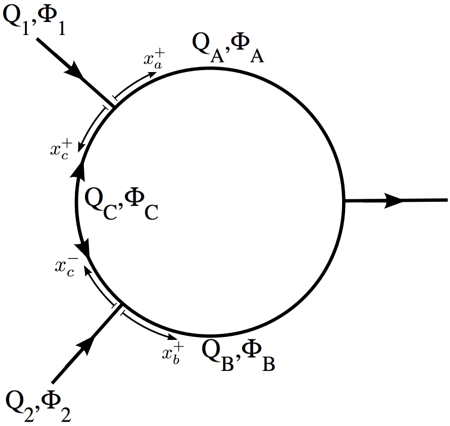

$$Q_C R_C + Q_B R_B \quad = \quad Q_A R_A$$

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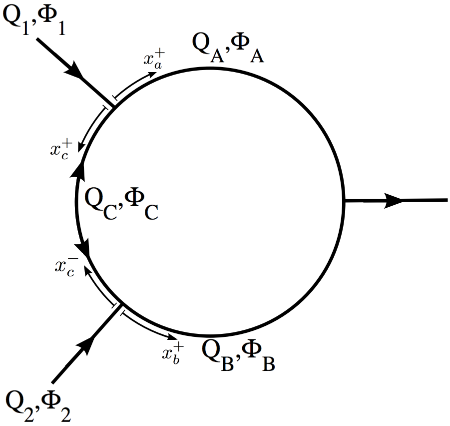

$$
\begin{aligned}
Q_C R_C + Q_B R_B &= Q_A R_A \\
Q_C &= \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C}
\end{aligned}
$$

the flow equation

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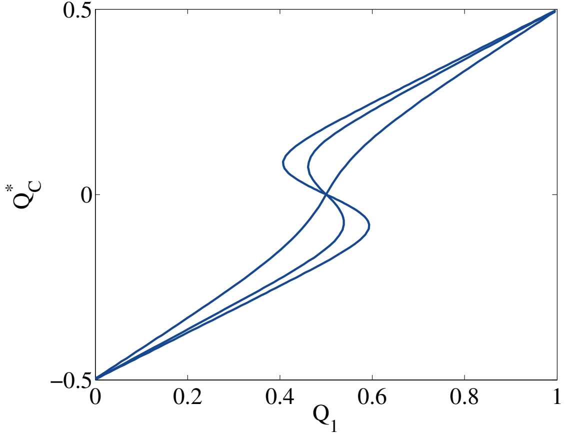
$$Q_C = \psi(Q_C)$$

multiple equilibria



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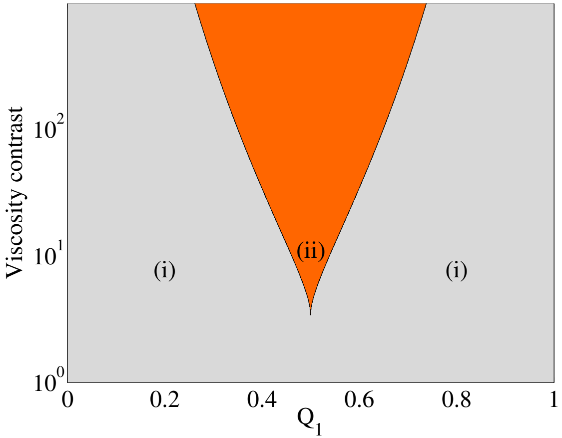

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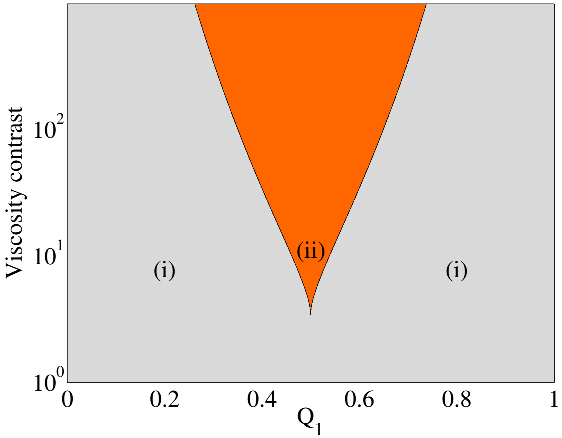

Boundary condition is $\dfrac{d\psi}{dQ_C} = 1.$

PDE model

For vessel i,

$$\frac{d\Phi_i}{dt} + \left(\frac{4Q_i(t)}{\pi d_i^2}\right)\frac{d\Phi_i}{dx_i} = 0.$$

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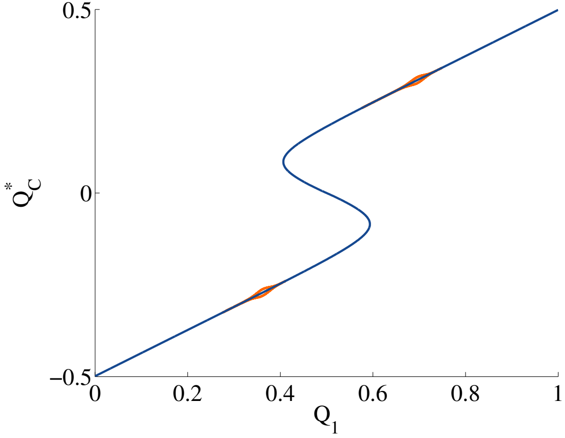
This methodology scales well to larger networks.

limit cycles



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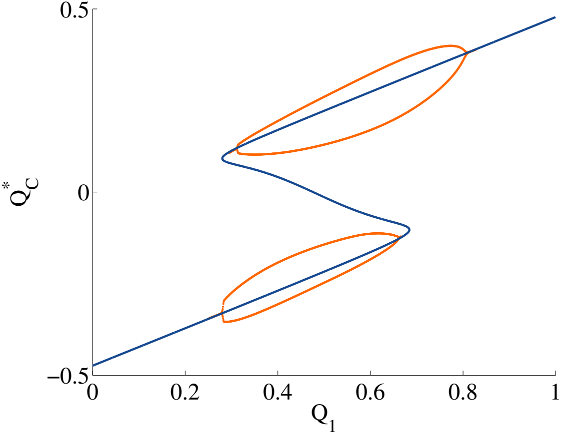
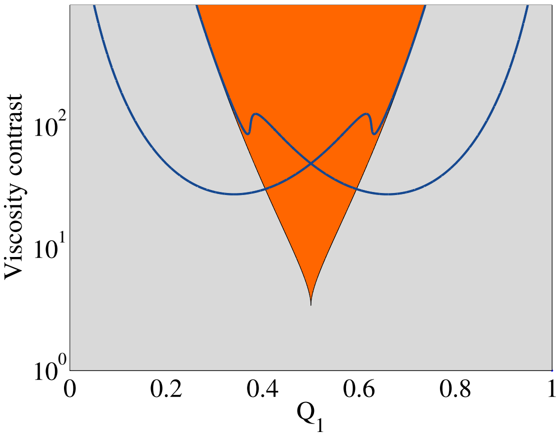


dynamics phase portrait



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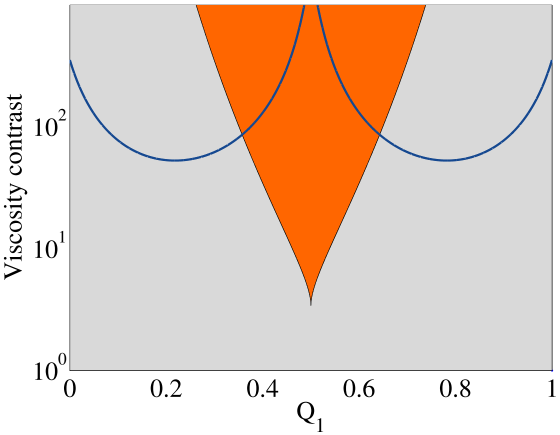

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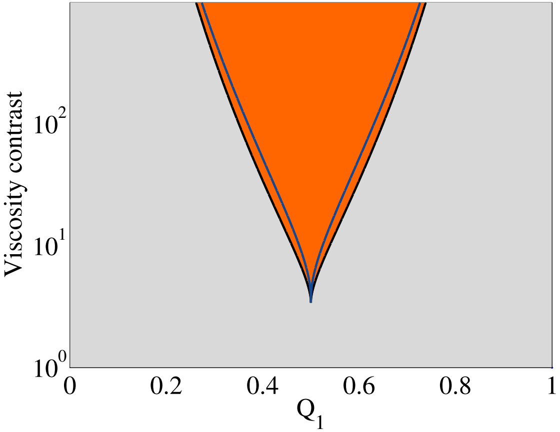

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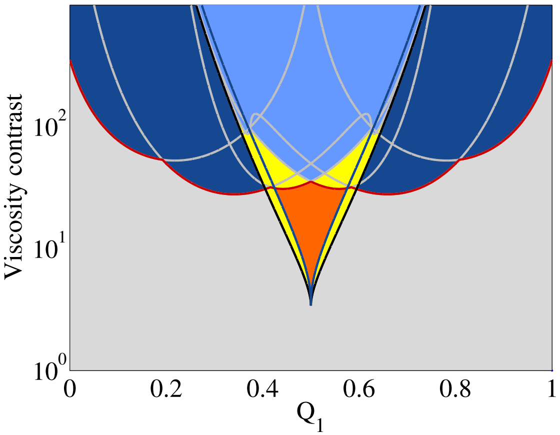

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- More interesting dynamics?

acknowledgements

- NSF (DMS-1211640)
- Babson College new faculty research fund