Spontaneous oscillations in simple fluid networks

Nathaniel Karst\textsuperscript{1}, John Geddes\textsuperscript{2}, & Brian Story\textsuperscript{2}

\textsuperscript{1}Babson College

\textsuperscript{2}F.W. Olin College of Engineering
In single capillaries the flow may become retarded or accelerated from no visible cause; in capillary anastomoses the direction of flow may change from time to time.

(1922) Krogh
big questions

- Are spontaneous oscillations possible in the absence of biologic control given biologically relevant fluid properties and network geometries?
- If so, what properties are necessary and/or sufficient to achieve oscillations?
- If not, are certain properties inhibiting oscillations? Topology? Geometry? Viscosity?
- Or is biologic control simply essential for interesting behaviors to emerge?
big questions

- Are spontaneous oscillations possible in the absence of biologic control given biologically relevant fluid properties and network geometries?
- If so, what properties are necessary and/or sufficient to achieve oscillations?
- If not, are certain properties inhibiting oscillations? Topology? Geometry? Viscosity?
- Or is biologic control simply essential for interesting behaviors to emerge?
big questions

▶ Are spontaneous oscillations possible in the absence of biologic control given biologically relevant fluid properties and network geometries?
▶ If so, what properties are necessary and/or sufficient to achieve oscillations?
▶ If not, are certain properties are inhibiting oscillations? Topology? Geometry? Viscosity?
big questions

- Are spontaneous oscillations possible in the absence of biologic control given biologically relevant fluid properties and network geometries?
- If so, what properties are necessary and/or sufficient to achieve oscillations?
- If not, are certain properties inhibiting oscillations? Topology? Geometry? Viscosity?
- Or is biologic control simply essential for interesting behaviors to emerge?
related work

(1994) Kiani et al. – simulations of large networks show oscillations in the absence of biologic control (≈ 400 vessel segments)
related work

(1994) Kiani et al. – simulations of large networks show oscillations in the absence of biologic control (≈ 400 vessel segments)

(2000) Carr and Lacoin – simulations show oscillations and period doubling dynamics in small networks (≈ 15 vessel segments)
related work

(1994) Kiani et al. – simulations of large networks show oscillations in the absence of biologic control (≈ 400 vessel segments)

(2000) Carr and Lacoin – simulations show oscillations and period doubling dynamics in small networks (≈ 15 vessel segments)

(2007) Geddes et al. – simple two-node network model predicts oscillations only for biologically irrelevant network geometries

(2012) Forouzan et al. – in vitro experiments with real blood exhibit oscillations in good agreement with theoretical work
related work

(1994) Kiani *et al.* – simulations of large networks show oscillations in the absence of biologic control (≈ 400 vessel segments)

(2000) Carr and Lacoin – simulations show oscillations and period doubling dynamics in small networks (≈ 15 vessel segments)

(2007) Geddes *et al.* – simple two-node network model predicts oscillations only for biologically irrelevant network geometries

(2012) Forouzan *et al.* – *in vitro* experiments with real blood exhibit oscillations in good agreement with theoretical work
example networks and assumptions

- Ignore growth and adaption
- Inlet and outlet pressures and flows are constant
- Incompressible, laminar flow
example networks and assumptions

- Ignore growth and adaption
example networks and assumptions

- Ignore growth and adaption
- Inlet and outlet pressures and flows are constant
example networks and assumptions

- Ignore growth and adaption
- Inlet and outlet pressures and flows are constant
- Incompressible, laminar flow
We approximate the viscosity of the fluid with an Arrhenius law

\[ \mu = \mu_\alpha \left( \frac{\mu_\beta}{\mu_\alpha} \right)^\phi. \]
We approximate the viscosity of the fluid with an Arrhenius law

\[ \mu = \mu_\alpha \left( \frac{\mu_\beta}{\mu_\alpha} \right)^\phi. \]

In the context of microvascular blood flow,
We approximate the viscosity of the fluid with an Arrhenius law

\[ \mu = \mu_\alpha \left( \frac{\mu_\beta}{\mu_\alpha} \right)^\phi. \]

In the context of microvascular blood flow,

- \( \mu_\alpha \) is the viscosity of plasma,
We approximate the viscosity of the fluid with an Arrhenius law

\[ \mu = \mu_\alpha \left( \frac{\mu_\beta}{\mu_\alpha} \right)^\phi. \]

In the context of microvascular blood flow,

- \( \mu_\alpha \) is the viscosity of plasma,
- \( \mu_\beta \) is the viscosity of red blood cells,
We approximate the viscosity of the fluid with an Arrhenius law

\[ \mu = \mu_\alpha \left( \frac{\mu_\beta}{\mu_\alpha} \right)^\phi. \]

In the context of microvascular blood flow,

- \( \mu_\alpha \) is the viscosity of plasma,
- \( \mu_\beta \) is the viscosity of red blood cells,
- \( \phi \in [0, 1] \) is the hematocrit, i.e., red blood cell concentration.
plasma skimming

(1982) Klitzman and Johnson
video

H.N. Mayrovitz

N. Karst
current network

\[ Q_C, \Phi_C \]

\[ Q_A, \Phi_A \]

\[ Q_B, \Phi_B \]

\[ Q_1, \Phi_1 \]

\[ Q_2, \Phi_2 \]
current network

\[ Q_C R_C + Q_B R_B = Q_A R_A \]
current network

\[ Q_C R_C + Q_B R_B = Q_A R_A \]

\[ Q_C = \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C} \]
Flow in branch $C$ depends on the hydraulic resistances, which depend on the viscosity in each branch through Poiseuille's law, which depends on the hematocrit in each branch through the Arrhenius law, which depends on the flow in each branch through plasma skimming.

\[
Q_C = \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C}
\]
the flow equation

\[ Q_C = \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C} \]

- Flow in branch C depends on the hydraulic resistances,
the flow equation

\[ Q_C = \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C} \]

- Flow in branch C depends on the hydraulic resistances,
- which depend on the viscosity in each branch through Poiseuille’s law,
the flow equation

\[ Q_C = \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C} \]

- Flow in branch C depends on the hydraulic resistances,
- which depend on the viscosity in each branch through Poiseuille’s law,
- which depends on the hematocrit in each branch through the Arrhenius law,
The flow equation

\[ Q_C = \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C} \]

- Flow in branch C depends on the hydraulic resistances,
- which depend on the viscosity in each branch through Poiseuille’s law,
- which depends on the hematocrit in each branch through the Arrhenius law,
- which depends on the flow in each branch through plasma skimming . . .
the flow equation

\[ Q_C = \frac{Q_1 R_A - Q_2 R_B}{R_A + R_B + R_C} \]

- Flow in branch C depends on the hydraulic resistances,
- which depend on the viscosity in each branch through Poiseuille’s law,
- which depends on the hematocrit in each branch through the Arrhenius law,
- which depends on the flow in each branch through plasma skimming . . .

\[ Q_C = \psi(Q_C) \]
multiple equilibria
multiple equilibria

Boundary condition is $d\psi = 1$. 

N. Karst JMM 2014
multiple equilibria

Boundary condition is $\frac{d\psi}{dQ_C} = 1$. 
PDE model

For vessel \( i \),

\[
\frac{d\Phi_i}{dt} + \left( \frac{4Q_i(t)}{\pi d_i^2} \right) \frac{d\Phi_i}{dx_i} = 0.
\]
PDE model

For vessel \( i \),

\[
\frac{d\Phi_i}{dt} + \left( \frac{4Q_i(t)}{\pi d_i^2} \right) \frac{d\Phi_i}{dx_i} = 0.
\]

Nondimensionalization using

\[
\hat{x}_i = \frac{x_i}{\ell_i},
\]

This methodology scales well to larger networks.
PDE model

For vessel $i$,

$$\frac{d\Phi_i}{dt} + \left( \frac{4Q_i(t)}{\pi d_i^2} \right) \frac{d\Phi_i}{d\hat{x}_i} = 0.$$ 

Nondimensionalization using

$$\hat{x}_i = \frac{x_i}{\ell_i}, \quad \hat{Q}_i = \frac{Q_i}{Q_{total}},$$

This methodology scales well to larger networks.

N. Karst JMM 2014
PDE model

For vessel $i$,

$$\frac{d\Phi_i}{dt} + \left( \frac{4Q_i(t)}{\pi d_i^2} \right) \frac{d\Phi_i}{dx_i} = 0.$$

Nondimensionalization using

$$\hat{x}_i = \frac{x_i}{\ell_i}, \quad \hat{Q}_i = \frac{Q_i}{Q_{total}}, \quad \hat{t} = \frac{Q_{total}}{V_{total}} t$$
PDE model

For vessel $i$,

$$\frac{d\Phi_i}{dt} + \left( \frac{4Q_i(t)}{\pi d_i^2} \right) \frac{d\Phi_i}{dx_i} = 0.$$ 

Nondimensionalization using

$$\hat{x}_i = \frac{x_i}{\ell_i}, \quad \hat{Q}_i = \frac{Q_i}{Q_{total}}, \quad \hat{t} = \frac{Q_{total}}{V_{total}} t$$

gives
PDE model

For vessel $i$,

\[ \frac{d\Phi_i}{dt} + \left( \frac{4Q_i(t)}{\pi d_i^2} \right) \frac{d\Phi_i}{dx_i} = 0. \]

Nondimensionalization using

\[ \hat{x}_i = \frac{x_i}{\ell_i}, \quad \hat{Q}_i = \frac{Q_i}{Q_{total}}, \quad \hat{t} = \frac{Q_{total}}{V_{total}} t \]

gives

\[ \frac{d\Phi_i}{d\hat{t}} + \left( \frac{Q_i V_{total}}{V_i Q_{total}} \right) \frac{d\Phi_i}{d\hat{x}_i} = 0. \]
PDE model

For vessel $i$,

$$
\frac{d\Phi_i}{dt} + \left( \frac{4Q_i(t)}{\pi d_i^2} \right) \frac{d\Phi_i}{dx_i} = 0.
$$

Nondimensionalization using

$$
\hat{x}_i = \frac{x_i}{\ell_i}, \quad \hat{Q}_i = \frac{Q_i}{Q_{total}}, \quad \hat{t} = \frac{Q_{total}}{V_{total}}t
$$

gives

$$
\frac{d\Phi_i}{d\hat{t}} + \left( \frac{Q_i V_{total}}{V_i Q_{total}} \right) \frac{d\Phi_i}{d\hat{x}_i} = 0.
$$

This methodology scales well to larger networks.
limit cycles
limit cycles
dynamics phase portrait
dynamics phase portrait
dynamics phase portrait

![Graph of viscosity contrast vs. Q1](image-url)
dynamics phase portrait

Viscosity contrast

$Q_1$
future work

- Empirical confirmation
future work

- Empirical confirmation
- Increase topological complexity to search for oscillations in biologically relevant networks
future work

- Empirical confirmation
- Increase topological complexity to search for oscillations in biologically relevant networks
- Spontaneous flow reversal?
future work

- Empirical confirmation
- Increase topological complexity to search for oscillations in biologically relevant networks
- Spontaneous flow reversal?
- More interesting dynamics?
acknowledgements

- NSF (DMS-1211640)
- Babson College new faculty research fund