QTM1300 Final

READ THESE INSTRUCTIONS BEFORE CONTINUING.

- Don't panic.
- You **may** work with others on this exam.
- Please **do not** go to the Math Resource Center for help on this exam. This is a courtesy to students who need help studying for their timed, in-class exams.
- **Show your work**. No work means no credit.
- For work in Excel, make each problem a different sheet in the same file and upload a single Excel document to Blackboard.
- There will be an oral exam related to this assessment. You must be able to explain every problem clearly and concisely in order to receive full credit.
- This is a take-home exam, and so there are very high standards of professionalism; make sure your final document is something that you would turn in to your boss.
- This test is due **in my office Babson 316** on **Monday 10 December 2012** at **noon**; this includes having any Excel document uploaded.
- **ONLY ATTEMPT PROBLEM 7 IF YOU ARE VERY CONFIDENT ABOUT PROBLEMS 1 – 6**

Once you've read and understood the instructions above, please print your name in the space below.

(1) Your FME group has decided to sell custom t-shirts that that draw attention to good causes, promote businesses and the like. As part of your market research, you're looking at the supply and demand trends exhibited by your competitors. Your team has assembled the data featured in Table 1.

Price per unit	**Supply**	**Demand**
0	0	70
5	0	21
10	3	15
15	9	4
20	11	2
30	16	0
35	24	0

Table 1: Supply and demand (hundreds of t-shirts) tracked by selling price (dollars) for a competitor's t-shirt company.

Based on your competitor's data, what is your best estimate of the the price your company should charge per shirt in order to maximize revenue?

(2) Imagine we have some data tracking the marginal revenue and marginal cost of our business. These data are featured in Figure 1 below.

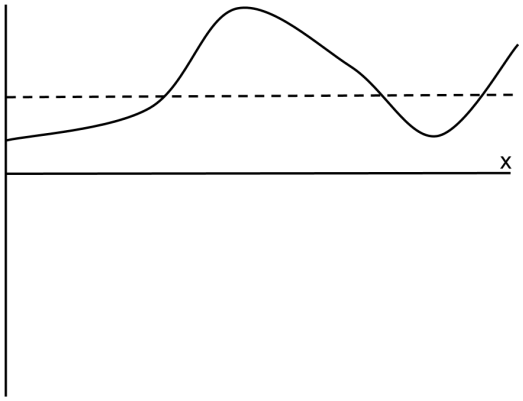
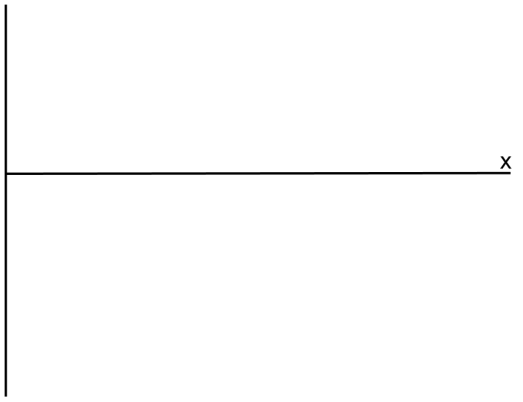
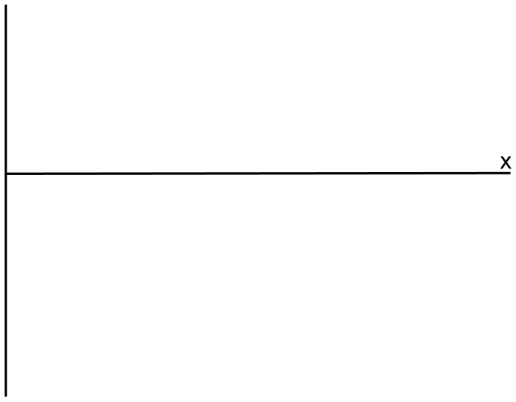


Figure 1: Marginal revenue (dashed line) and marginal cost (solid line) as a function of number of units sold

(a) Imagine that marginal revenue and marginal cost are given by the graph in Figure 1. On the blank axes below, sketch the marginal profit.



(b) Imagine that marginal revenue and marginal cost are given by the graph in Figure 1. On the blank axes below, sketch the total profit. Clearly indicate the global maximum of the total profit function.



(c) Now imagine that we have algebraic expressions for both the marginal revenue and marginal cost: $R'(x) = 8$, $C'(x) = x^2 - 2x + 6$. How many units should you sell in order to maximize total profit?

(d) Now imagine that we have algebraic expressions for both the marginal revenue and marginal cost: $R'(x) = 8$, $C'(x) = x^2 - 2x + 6$. What is the total profit function if the fixed cost is 12?

(3) After leaving Babson, you and some fellow alums launched a new microbrewery venture. After resounding success, you've opened two main brewing facilities, one in South Boston and the other in Windsor, Vermont. As an entrepreneur, you're both CEO and production/distribution coordinator. Your responsibilities in the second role are twofold. First, you must set the output, measured in barrels of beer, of each brewing facility. Second, you must assign each barrel produced to one of your two main markets: Boston and New York. You know from historical data that the Boston market orders on average 500 barrels per month, while the New York market orders 530 barrels per month. You also know that your facilities are not identical: the flagship facility in South Boston can produce up to 620 barrels per month, while the satellite facility in Windsor can only produce up to 410 barrels per month. Furthermore, you have some transportation cost data which is summarized in Table 2

Origin	**Destination**	**Cost**
Windsor	Boston	10
Windsor	New York	25
South Boston	Boston	5
South Boston	New York	15

Table 2: Transportation cost ($/barrel) from each brewing facility to each market

Your total monthly transportation budget is $5,550. In order to maximize your market share, you'd like to ship as many barrels as possible. What production levels and distribution orders do you send to each brewing facility?

(4) Assume that the total national consumption in the country of Andorra is given by the function $C(x)$, where x here represents the total national income of Andorra. Then the total national savings is given by $S(x) = x - C(x)$. Economists call the derivatives of $C(x)$ and $S(x)$ the *marginal propensity to consume* and the *marginal propensity to save*, respectively.

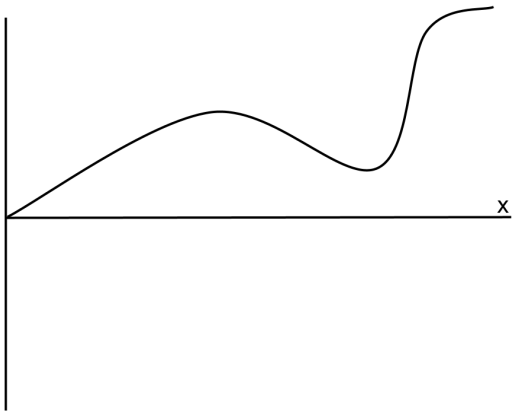
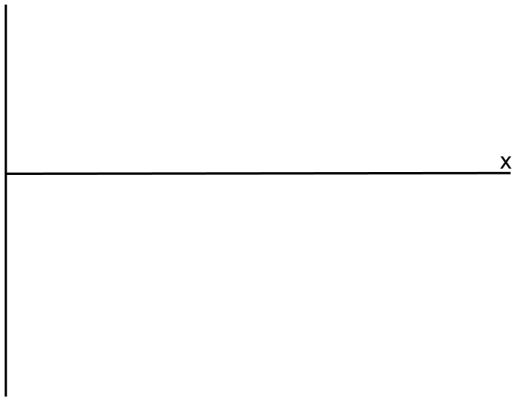


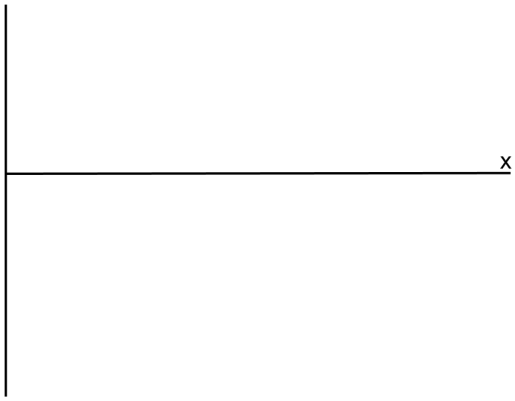
Figure 2: Total national consumption as a function of total national income.

(a) Imagine that $C(x)$ is given by the graph in Figure 2. On the blank axes below, sketch the marginal propensity to consume.

(b) Imagine that $C(x)$ is given by the graph in Figure 2. On the blank axes below, sketch the marginal propensity to save.



(c) Imagine that $C(x)$ is given by the graph in Figure 2. On the blank axes below, sketch the total national savings.



(5) You own a ice cream stand on a popular Maine beach. You have been collecting data for the past 10 years about your company's performance, and you're looking to figure out what exactly is driving sales growth. You have two main hypotheses. The first is that global warming is causing people to want a cool treat more frequently. The second is that the median household income in town in which your stand is located is increasing which might imply that people on the beach have more disposable income. To get a handle on figuring out which of these hypotheses is better, you've collected the data featured in Table 3.

Year	**Total Sales**	**Average Temp.**	**Median Income**
2000	27.93	86.92	30.11
2001	28.29	88.51	31.48
2002	29.70	88.01	32.03
2003	31.09	87.05	33.34
2004	31.12	90.22	34.20
2005	30.12	85.52	35.26
2006	31.83	87.45	36.36
2007	34.61	85.85	37.43
2008	37.37	86.14	38.33
2009	37.80	87.88	39.31

Table 3: Data collected for the total sales (thousands of dollars), average temperate (degrees Fahrenheit), and median household income (thousands of dollars) for June of the indicated year

Restricting your analysis to linear models, is temperature or median income a better predictor of your sales?

(6) You might remember me claiming in class that going to grad school never pays for itself. Let's dive into this a little further. Imagine we have two college friends of the same age who graduate together in 2013. One goes straight to graduate school and becomes the Graduate Student, and the second goes straight into the workforce and becomes the Young Professional. Being responsible people, both immediately start thinking about funding their retirement through a 401(k) account. Federal law limits contributions to 401(k) retirement accounts to $17,000 annually. Unfortunately, the average graduate student (if they make at money at all) makes less than $20,000 a year. As you might expect, this means that the Graduate Student doesn't save for retirement at all while still in school. Let's compare this to the Young Professional who is saving the full $17,000 each year straight out of college in 12 equally sized monthly contributions. To keep things fair and simple, let's assume that both the Graduate Student and the Young Professional are contributing to accounts that provide a steady 5% annual rate of return and compound monthly.

(a) Graduate school typically takes anywhere from 5 years to 10 years. What is the *best* case for how much the Graduate Student will be behind the Young Professional in retirement account value when both are 65? What is the *worst* case?

(b) Imagine the best case happens from the subproblem above. If upon retirement at age 65, both the Gradaute Student and the Young Professional roll their 401(k) accounts over into a 20 year annuity returning 5% annually and compounding monthly, how much more would the Young Professional withdraw per month than the Graduate Student?

(c) Imagine that the Graduate Student chooses not to open a 401(k) account, but instead opts for a normal compound interest investment returning 6% interest and compounding monthly. The good news here is that a normal compound interest investment does not have contributions limits like the 401(k) does. If the Graduate Student decides to make equally sized monthly contributions after finishing graduate school and up to retirement, how large would this monthly contribution need to be in order to catch up to the Young Professional in terms of retirement savings by age 65?

(7) Your bakery has a major piece of equipment that depreciates at a continuous rate of $d(t)$ dollars per month, where t is the number of months since the last overhaul of the machine. You'd think that overhauling the machine very frequently would be a good thing to keep its value high, but you have to take into consideration that each time you overhaul the machine, you incur a total cost of A in parts, labor, and lost production time. Naturally, you want to determine the optimal time between overhauls.

(a) What does $\int_0^t d(s)ds$ represent?

(b) Let the function $C(T)$ be defined by

$$C(t) = \frac{1}{t}\left(A + \int_0^t d(s)ds\right).$$

What does $C(t)$ represent? Why would the company want to minimize $C(t)$?

(c) Show that you should overhaul the machine after T months if and only if when $d(T) = C(T)$.