On Transceiver Signal Linearization and the Decoding Delay of Maximum Rate Complex Orthogonal Space-Time Block Codes

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***Abstract*—Complex orthogonal designs (CODs) have been successfully implemented in wireless systems as complex orthogonal space-time block codes (COSTBCs). Certain properties of the underlying CODs affect the performance of the codes. In addition to the main properties of a COD's rate and decoding delay, a third consideration is whether the COD can achieve transceiver signal linearization, a property that facilitates practical implementation by, for example, significantly simplifying the receiver structure for iterative decoding. It has been shown that a COD can achieve this transceiver signal linearization if the nonzero entries in any given row of the matrix are either all conjugated or all nonconjugated. This paper determines the conditions under which maximum rate CODs can achieve this desirable property. For an odd number of transmit antennas, it is shown that maximum rate CODs that achieve the lower bound on decoding delay can also achieve transceiver signal linearization. In contrast, for an even number of transmit antennas, maximum rate CODs that achieve the lower bound on delay cannot achieve this linearization. In this latter case, linearization is possible only if the COD achieves at least twice the lower bound on delay. This work highlights the tradeoffs among these three important properties.**

***Index Terms*—Complex orthogonal designs (CODs), iterative decoding, space-time block codes, transceiver signal linearization.**

I. Introduction

We define an $[r, n, k]$ complex orthogonal design (COD) as an $r \times n$ matrix $\mathbf{G}$ with entries from $\{0, \pm z_1, \ldots, \pm z_k, \pm z_1^*, \ldots, \pm z_k^*\}$ such that $\mathbf{G}^H\mathbf{G} = \sum_{l=1}^{k} |z_l|^2 \mathbf{I}_n$, where H is the Hermitian transpose and $\mathbf{I}_n$ is the $n \times n$ identity matrix [1], [2]. Geramita and Seberry provide a comprehensive review of classical orthogonal designs [3], and Liang reviews and defines their generalizations [4]. Examples of the type of CODs considered in this paper can be found readily in the literature (e.g., [2], [5]). Such CODs have been applied as complex orthogonal space-time block codes (COSTBCs), which are useful in wireless applications due to their simple maximum-likelihood decoding rule and their guarantee of full diversity [1].

Several factors must be considered when choosing the underlying COD for application in a practical coding system. The rate (i.e., the ratio of the number of variables to the number of rows) and the minimum decoding delay (i.e., the minimum number of rows) for a given rate are two of the fundamental considerations. A third consideration is whether the COD allows for a linearized description of the received signal, as achieved by the original Alamouti code [6]. In his fundamental paper, Alamouti [6] used the following underlying COD:

$$\mathbf{A} = \begin{pmatrix} z_0 & z_1 \\ -z_1^* & z_0^* \end{pmatrix}. \tag{1}$$

Alamouti showed that for a flat-fading channel that is assumed constant over two consecutive symbol times, the received signal for his transmit diversity system can be expressed as

$$\begin{aligned} r_0 &= r_t = h_0 z_0 + h_1 z_1 + n_0 \\ r_1 &= r_{t+T} = -h_0 z_1^* + h_1 z_0^* + n_1 \end{aligned}$$

where r_0 and r_1 are the received signals at times t and $t+T$, respectively, n_0 and n_1 are complex receiver noises, and h_0 and h_1 are the complex channel gain coefficients. Therefore, the receiver can be based on the use of a linear combiner producing two combined signals

$$\begin{aligned} \tilde{z}_0 &= h_0^* r_0 + h_1 r_1^* \\ \tilde{z}_1 &= h_1^* r_0 - h_0 r_1^* \end{aligned}$$

which are then sent to the maximum likelihood detector for decoupled decoding. The full concept of the process is described in detail in Haykin and Moher's textbook [7]. Alamouti showed that this process can be easily extended to multiple receive antennas [6], and Lu and Wang further extended the approach to multiuser scenarios [8]. In general, the same receiver structure based on a linear combiner can be applied if the received signal vector $\mathbf{y}$ has the following form:

$$\mathbf{y} = \mathbf{Hz} + \mathbf{n}$$

where the elements of the vector $\mathbf{y}$ are either samples of the received signal or their conjugates, the $r \times n$ matrix $\mathbf{H}$ contains complex linear combinations of channel state coefficients and their conjugates, and the ith component of $\mathbf{n}$ represents the

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equivalent noise effect observed in time slot i, $i = 1, 2, \ldots, r$ [9].

This latter form is often referred to in the literature (e.g., [8]–[11]) as a linearized description of the transceiver signal, and we say that the code achieves transceiver signal linearization. This description allows for the application of a linear combiner before a maximum likelihood detector. This significantly simplifies the receiver structure, particularly for iterative decoding such as in the receivers studied by Lu and Wang [8] and the RAKE-based receivers studied by Jayaweera and Poor [10].

Su, Batalama, and Pados recently determined the conditions under which a COSTBC permits such a linearized received signal expression [9]. Specifically, they showed that if the underlying COD $\mathbf{G}$ can be arranged so that the nonzero entries in any given row are either all conjugated or all nonconjugated, then $\mathbf{G}$ can achieve this linearization [9]. If $\mathbf{G}$ achieves this property, we say that it is *conjugation-separated*. Su *et al.* also noted that the tranceiver signal linearization is important because it allows for backwards compatibility with a variety of signal processing techniques and for the design of certain low complexity filters and equalizers [9].

Su *et al.* focused on determining when square CODs can achieve conjugation-separation, hence transceiver signal linearization [9]. Unfortunately, they determined that the rate of such square CODs approaches zero as the number of columns (i.e., antennas) increases [9]. Since a higher rate is preferred in practice, their work motivated us to determine the conditions under which rectangular CODs of higher rates, namely of maximum rate, can achieve transceiver signal linearization. Liang determined that the maximum rate for a COD with $2m-1$ or $2m$ columns is $R_0 = \frac{m+1}{2m}$ [2], so our focus is to determine the conditions under which a COD with this maximum rate can achieve transceiver signal linearization. Since these maximum rate CODs are no longer square, as opposed to the square CODs considered by Su *et al.* [9], we must also aim to minimize their decoding delay. Adams *et al.*. determined that a lower bound on decoding delay for maximum rate CODs with $2m-1$ or $2m$ columns is $r_0 = \binom{2m}{m-1}$ [12], which is achievable if the number of columns is congruent to 0, 1, or 3 modulo 4, while the best achievable delay when the number of columns is congruent to 2 modulo 4 is $2r_0 = 2\binom{2m}{m-1}$ [13].

In this paper studying the tradeoffs among transceiver signal linearization, rate, and decoding delay, we show that maximum rate R_0 CODs that achieve the lower bound r_0 on delay can achieve transceiver linearization if and only if the number of columns is odd. When the number of columns is even, a maximum rate COD must achieve at least twice the lower bound on delay (which is the best possible delay when the number of columns is 2 modulo 4 [13]) in order to achieve this transceiver signal linearization.

II. Preliminaries

For any COD $\mathbf{G}$ with $2m-1$ or $2m$ columns, and for any fixed z_i, $1 \leq i \leq k$, Liang showed that the orthogonality constraint implies that it is possible to transform $\mathbf{G}$ through equivalence operations (e.g., rearrange order of rows/columns, negate rows/columns, conjugate/negate all instances of a given variable) so that the following submatrix $\mathbf{B}_i$, which contains all instances of the variable z_i (up to conjugation and sign), appears within the top $2m-1$ or $2m$ rows, respectively, [2]

$$\mathbf{B}_i = \begin{pmatrix} z_i\mathbf{I} & \mathbf{M}_i \\ -\mathbf{M}_i^H & z_i^*\mathbf{I} \end{pmatrix}.$$

If $\mathbf{G}$ is of maximum rate, Liang proved several properties concerning the $\mathbf{B}_i$ and $\mathbf{M}_i$ submatrics [2]. For example, he showed that if a maximum rate $\mathbf{G}$ has $2m$ columns, then $\mathbf{M}_i$ has order $m \times m$. If $\mathbf{G}$ has $2m-1$ columns, then $\mathbf{M}_i$ is either $(m-1)\times m$ or $m \times (m-1)$. In this case, we will assume that $\mathbf{M}_i$ is $m \times (m-1)$; all proofs can be altered slightly for the alternative. He also showed that these $\mathbf{M}_i$ submatrices have no zero entries [2].

Let any (possibly noncontiguous) 2×2 orthogonal submatrix of a COD that is isomorphic under equivalence operations to Alamouti's original COD (see (1)) be called an *Alamouti* 2×2. We say that two rows of a COD *share* an Alamouti 2×2 over two columns if the intersection of these rows and columns forms such a 2×2 orthogonal submatrix.

We now recall a simple but powerful result originally proven in Part 1) of Lemma 3.2 in [13]:

Result 2.1: [13] Let $\mathbf{r}_s$ and $\mathbf{r}_t$ be distinct rows of an $[r_0, n, r_0R_0]$ COD $\mathbf{G}$. Then, rows $\mathbf{r}_s$ and $\mathbf{r}_t$ share an Alamouti 2×2 over columns $\mathbf{c}_h$ and $\mathbf{c}_i$ if and only if $\mathbf{r}_s$ and $\mathbf{r}_t$ are simultaneously nonzero exactly in columns $\mathbf{c}_h$ and $\mathbf{c}_i$ and never simultaneously zero in any column.

III. Transceiver Linearization of Maximum Rate COSTBCs

In this section, we prove our main result by determining when a maximum rate COD can achieve conjugation-separation (as described above to mean that in any given row, the nonzero entries are either all conjugated or all nonconjugated). Then, the recent work by Su *et al.* [9] implies that such CODs can be implemented to achieve transceiver signal linearization. This linearization is beneficial for practical systems, particularly those involving iterative decoding [8], [10].

Theorem 3.1: Let $\mathbf{G}$ be a maximum rate $[r_0, 2m-1, r_0R_0]$ COD that achieves the lower bound on decoding delay. Then, $\mathbf{G}$ is equivalent to a COD that is conjugation-separated, and, therefore, this arrangement of $\mathbf{G}$ can achieve transceiver signal linearization.

Proof: We can perform suitable equivalence operations to create the $\mathbf{B}_1$ submatrix for variable z_1 (see Section II). The first m rows of $\mathbf{B}_1$ each contain $m-1$ zeros and one entry of z_1, and the last $m-1$ rows of $\mathbf{B}_1$ each contain $m-2$ zeros and one entry of z_1^*. Since all instances of z_1 (up to conjugation and sign) appear within $\mathbf{B}_1$, all instances of z_1 contained in rows with $m-1$ zeros are nonconjugated, and all instances of z_1 contained in rows with $m-2$ zeros are conjugated.

Now, without affecting the conjugation of the instances of z_1, we can perform equivalence operations on $\mathbf{G}$ to move from displaying $\mathbf{B}_1$ to displaying another $\mathbf{B}_j$ submatrix for $j \neq 1$ [2], [12]. Every instance of z_j (up to conjugation and sign) will appear within this $\mathbf{B}_j$ submatrix [2], [12]. By known properties of $\mathbf{B}_j$ (see Section II), it is possible to conjugate all instances of z_j so that the instances of z_j within rows with $m-1$ zeros are nonconjugated and the instances of z_j within rows with $m-2$ zeros are conjugated.

We can repeat this procedure by rearranging $\mathbf{G}$ to display $\mathbf{B}_\ell$ for each $1 \le \ell \le r_0R_0$. When a specific $\mathbf{B}_\ell$ is formed, it may be required to conjugate all instances of z_ℓ so that rows with $m-1$ zeros have nonconjugated versions of z_ℓ and rows with $m-2$ zeros have conjugated versions of z_ℓ. Since every instance of an arbitrary variable z_ℓ appears within $\mathbf{B}_\ell$, and since this variable only undergoes possible conjugation when $\mathbf{G}$ is arranged to display the $\mathbf{B}_\ell$ submatrix, this procedure ensures that any variable within a row with $m-1$ zeros is nonconjugated and any variable within a row with $m-2$ zeros is conjugated. Hence, $\mathbf{G}$ is equivalent to a COD that achieves conjugation-separation and, hence, transceiver signal linearization. ∎

Theorem 3.2: Let $\mathbf{G}$ be a maximum rate $[r_0, 2m, r_0R_0]$ COD that achieves the lower bound on decoding delay. Then no arrangement of $\mathbf{G}$ is conjugation-separated.

Proof: We recall for clarity that a COD with $2m$ columns for m odd cannot achieve the decoding delay of r_0 [13], so the conditions of this theorem imply that m is even. Now, assume for contradiction that $\mathbf{G}$ is arranged to be conjugation-separated.

We have previously shown [13] that it is possible to select $2m$ rows of $\mathbf{G}$ such that for each $1 \le i \le 2m-1$, row $\mathbf{r}_i$ and row $\mathbf{r}_{i+1}$ share an Alamouti 2×2 over columns $\mathbf{c}_i$ and $\mathbf{c}_{2m}$. Furthermore, rows $\mathbf{r}_1, \ldots, \mathbf{r}_{2m-1}$ will be distinct, while $\mathbf{r}_{2m} = \mathbf{r}_1$.

We begin with row $\mathbf{r}_1$ of the following form, and since $\mathbf{G}$ is assumed to be conjugated-separated, we can assume that every entry z represents a nonconjugated variable (positive or negative)

$$\mathbf{r}_1 : z \quad 0 \quad z \quad 0 \quad \ldots \quad z \quad 0 \quad z \quad z.$$

Then, row $\mathbf{r}_2$ is selected as a row that shares an Alamouti 2×2 with row $\mathbf{r}_1$ over columns $\mathbf{c}_1$ and $\mathbf{c}_{2m}$. Thus, its entries in columns $\mathbf{c}_1$ and $\mathbf{c}_{2m}$ must be conjugated to ensure the orthogonality of the Alamouti 2×2. Then, by our assumption of conjugation-separation, the remaining nonzero entries of $\mathbf{r}_2$ will also be conjugated. Thus, by Result 2.1, $\mathbf{r}_2$ has the following form:

$$\mathbf{r}_2 : z^* \quad z^* \quad 0 \quad z^* \quad \ldots \quad 0 \quad z^* \quad 0 \quad z^*$$

where z^* represents any conjugated variable (positive or negative).

Iterating this procedure allows us to start with the given $\mathbf{r}_1$ and produce $\mathbf{r}_2, \ldots, \mathbf{r}_{2m}$. To maintain conjugation-separation, to form the requisite Alamouti 2×2 s between rows $\mathbf{r}_i$ and $\mathbf{r}_{i+1}$, and since row $\mathbf{r}_1$ is assumed to be nonconjugated, all rows with odd subscript are nonconjugated and all rows with even subscript are conjugated. So, in particular, $\mathbf{r}_1$ is nonconjugated and $\mathbf{r}_{2m}$ is conjugated. But this contradicts our earlier result that $\mathbf{r}_1 = \mathbf{r}_{2m}$. Hence, this contradiction shows that a maximum rate COD with an even number of columns cannot simultaneously achieve the lower bound on decoding delay and conjugation-separation. ∎

Theorem 3.3: It is possible to construct a maximum rate COD with any even number of columns that simultaneously achieves conjugation-separation and twice the lower bound on decoding delay.

Proof: This proof relies on the work of other authors. We first note that Su *et al.* use conjugation-separation (though not referred to as such) as a fundamental characteristic of their construction technique [5]. Their algorithm generates CODs with any even number of columns that achieve maximum rate, conjugation-separation, and twice the lower bound on decoding delay. Additionally, Liang's algorithm [2] generates maximum rate CODs with an even number of columns that achieve twice the lower bound on decoding delay. Steps 3) and 4) of his algorithm ensure that his examples are conjugation-separated (though, again, not referred to as such). These algorithms show that it is possible for a maximum rate COD with any even number of columns to simultaneously achieve conjugation-separation and twice the lower bound on decoding delay. As no maximum rate COD can achieve a delay that is between the lower bound and twice the lower bound [12], this is the best we can do in terms of delay under these restrictions. ∎

TABLE I
SUMMARY OF RESULTS ON THE DELAY AND TRANSCEIVER SIGNAL LINEARIZATION (TSL) OF MAXIMUM RATE CODs WITH UP TO TEN COLUMNS

Number of Columns	Decoding Delay	TSL	Minimum Achievable Delay
2	2	Yes	Yes
3	4	Yes	Yes
4	4	No	Yes
4	8	Yes	No
5	15	Yes	Yes
6	30	Yes	Yes
7	56	Yes	Yes
8	56	No	Yes
8	112	Yes	No
9	210	Yes	Yes
10	420	Yes	Yes

TABLE II
GENERAL RESULTS INDICATING WHEN MAXIMUM RATE, MINIMUM DECODING DELAY CODs CAN ACHIEVE TRANSCEIVER SIGNAL LINEARIZATION (TSL)

Number of Columns	Minimum Achievable Delay	TSL
4ℓ	$\binom{4\ell}{2\ell-1}$	No (Theorem 3.2)
$4\ell+1$	$\binom{4\ell+2}{2\ell}$	Yes (Theorem 3.1)
$4\ell+2$	$2\binom{4\ell+2}{2\ell}$	Yes (Theorem 3.3)
$4\ell+3$	$\binom{4\ell+4}{2\ell+1}$	Yes (Theorem 3.1)

IV. CONCLUSIONS

This paper determines the conditions under which maximum rate COSTBCs can simultaneously achieve transceiver signal linearization and optimal decoding delay. A COSTBC achieves such linearization when the underlying COD is conjugation-separated [9]. For any odd number of columns, we showed that there exist maximum rate CODs that simultaneously achieve conjugation-separation and the lower bound on decoding delay. For any even number of columns, a maximum rate COD that achieves the lower bound on delay cannot achieve conjugation-separation; however, for any even number of antennas, there exists a conjugation-separated COD that achieves twice the lower bound on delay. In the case when the number of columns is 2 modulo 4, achieving twice the lower bound on decoding delay is the best possible delay, while in the case of 0 modulo 4, this is twice the best achievable delay. These results are summarized in Table I for up to ten columns, while Table II summarizes the general results.

These results contribute to the understanding of the tradeoffs that must be made when considering the optimization and implementation of COSTBCs. Of course, it is only for two antennas that we can simultaneously achieve full rate, square size (meaning lowest possible delay), and transceiver signal linearization. As the number of antennas increases, there is a sacrifice in some or all of these parameters.

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